UDC 551.558.21:551.511.32:551.507.321.2:551.507.352(237.1)(788)

THE DYNAMIC STRUCTURE OF LEE WAVE FLOW AS OBTAINED FROM BALLOON AND AIRPLANE OBSERVATIONS

I. VERGEINER[1] and D. K. LILLY

National Center for Atmospheric Research,[2] Boulder, Colo.

ABSTRACT

The lee flow disturbances produced by the Front Range of the Colorado Rockies have been quantitatively observed in a continuing program at the National Center for Atmospheric Research. The results of midtroposphere constant-volume ballon and aircraft flights in the winter of 1966/67 are here presented. The relative merits and limitations of the two methods are compared with respect to various operational and inherent phenomenological difficulties of the subject. The nonstationarity of many flow features is inescapable and poses serious problems for data evaluation and theory. Schematically, we distinguish between smooth, wavy, and hydraulic jump-type flow patterns, but also observe some cases that do not fit well into any of these categories. The stronger stationary wave features can be compared with the "stable" resonance modes computed from stationary linear theory, that is, those modes which are insensitive to small changes in the upstream flow. The frequent occurrence of erratic and nonstationary flows may relate to the frequent existence of "unstable" or sensitive modes in the linear theory predictions. Examples of smooth and hydraulic jumplike flows are also shown and qualitatively compared to current theoretical predictions. Some suggestions are made for improvement of observational techniques in the downslope boundary layer.

1. INTRODUCTION

The Front Range of the central Colorado Rocky Mountains has been recognized as one of the most frequent sources in the United States of large amplitude lee waves and associated aircraft hazards. The establishment of the National Center for Atmospheric Research in the approximate center of this active region at Boulder has provided a remarkable opportunity for detailed and continued observational study of these phenomena. In addition, new instrumental platforms and techniques suitable for quantitative investigation of lee wave dynamics have been developed since the time of the Sierra (Bishop) wave investigation, but these have apparently not been systematically applied to high-amplitude lee wave situations. For reasons of scientific curiosity, and in order to test some of the new methods, an observational study was commenced in the winter of 1965–66 and is continuing. The purpose of the study is to obtain records of tropospheric and stratospheric kinematics and dynamics during typical and extreme conditions of westerly flow across the Continental Divide. This paper contains descriptions and analyses of the results obtained in 1966–67. In February 1968, a joint observational program on lee wave phenomena was carried out with participation by a number of United States and Canadian agencies. An account of the basic methods and early results of the program in 1968 has been published by Kuettner and Lilly (1968) and Lilly and Toutenhoofd (1969).

Figure 1 shows height contours and the terrain features of the Front Range area. For a distance of about 40 km, the Continental Divide consists of a well-defined north-south ridge with a nearly uniform elevation of about 3700 m. The foothills extend about 27 km to the east, terminating in a sharp dropoff at the longitude of Boulder. Eastward lie the flat high plains at an elevation of 1500 m, and to the west the Fraser and Colorado River Valleys, with elevations near 2500 m, separate the Front Range from a rather complex system of mountains, plateaus, and valleys, but with only isolated peaks above 3000 m. Figures subsequent to figure 1 show the contour profile of the region, averaged over a north-south distance of 20 km between the latitudes of Boulder and Longmont. By comparison with the Bishop wave case, the total altitude drop is about the same, but here it is somewhat more gradual and not confined by another range to the east. In a comparative study of lee waves and associated turbulence in the United States, Harrison and Sowa (1966) and Reiter and Foltz (1967) state that next to the Bishop wave the Front Range wave is the strongest, and that the latter is first in frequency of turbulence reported by United Air Lines pilots.

2. CONSTANT-VOLUME BALLOONS

Booker and Cooper (1965) pioneered in the use of constant-volume superpressured balloons for determination of three-dimensional trajectories in mountain wave conditions. In their study over the Pennsylvania Alleghenies, Mylar balloons of volume 0.32 m^3 with a small radar reflector were inflated so that they would float stably at a predetermined altitude and then were released at or near that altitude from either a tow balloon or an aircraft. The balloons were tracked by ground-based radar, with the aid of small radar reflectors or aluminized skin. When such balloons are displaced from their equilibrium floating altitude by a vertical air motion component, they tend to return to that altitude and to undergo a damped free oscillation about it. For such small balloons the restoring velocity is, however, generally less than 1 m sec^{-1} and therefore considerably smaller than typical vertical currents in well-developed lee waves. The question of how well the balloons follow the air motion is rather important. An attempt was made to confirm Booker and Cooper's (1965) computations in the following way.

[1] On leave from the Department of Meteorology and Geophysics, University of Innsbruck, Austria.

[2] The National Center for Atmospheric Research is sponsored by the National Science Foundation.

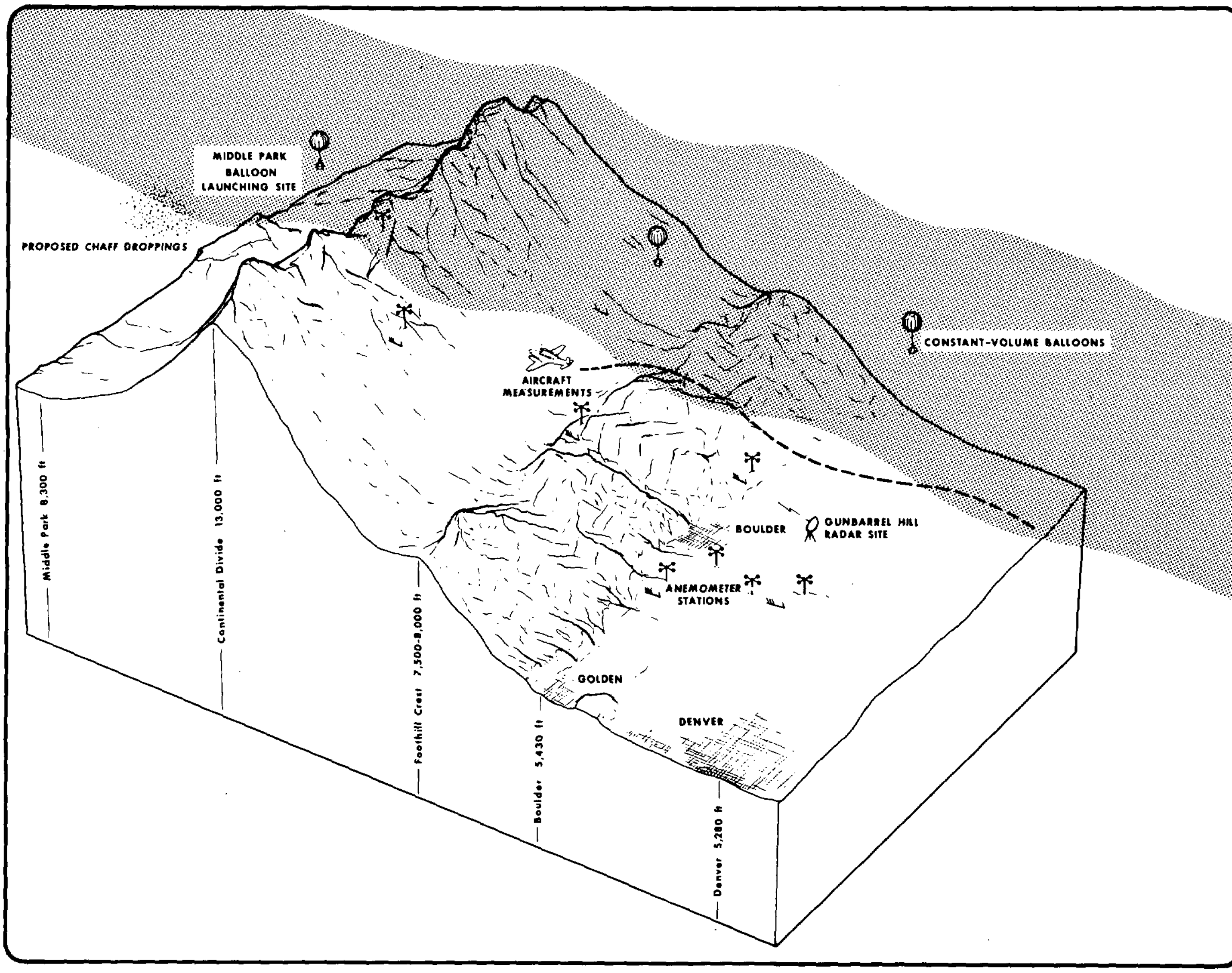

FIGURE 1.—Block diagram of the mountain wave study area in the central Colorado Rocky Mountains.

The vertical component of the equation of motion for the balloon is

$$\underset{\text{(weight)}}{-M\ddot{z}_B-Mg}+\underset{\text{(lift)}}{V\rho g}+\underset{\text{(drag)}}{C_DA\rho(W_A-W_B)|W_A-W_B|}=0 \quad (1)$$

where

M=total mass of balloon+transponder, 400 gm,
V=volume of balloon, 0.70 m^3,
A=cross section of balloon, 1 m^2,
C_D=drag coefficient, taken as 0.2,
ρ=density of ambient air, 750 gm m^{-3},
$z_B(t)$=height of balloon as a function of time,
$W_B=\dot{z}_B$=vertical velocity of balloon, and
W_A=vertical velocity of air.

Here equilibrium density ρ_e corresponding to the balloon's equilibrium floating level z_e is determined by $M=V\rho_e$. Rewriting (weight+lift) as $gV(\rho-\rho_e)$ and using $\Delta\rho\approx-10^{-4}$ $(m^{-1})\bar{\rho}\Delta z$ yields

$$(W_A-W_B)|W_A-W_B|=C_1\ddot{z}_B+C_2(z_B-z_e) \quad (2)$$

where $C_1=M/\bar{\rho}C_DA$ and $C_2=gV10^{-4}/C_DA$. With the approximate numerical values given above, $C_1\approx 2.67$ m and $C_2=0.0033$ m sec^{-2}. The period of free oscillation around the equilibrium level $2\pi\sqrt{C_1/C_2}\approx 3$ min is uncomfortably close to typical lee wave periods, but the oscillation is strongly damped.

Equation (2) was used in the form

$$\frac{d}{dt}(z_A-z_B)=\begin{cases}+\sqrt{+\text{right-hand side}} & \text{if RHS.}>0\\ -\sqrt{-\text{right-hand side}} & \text{if RHS.}<0\end{cases}$$

and integrated to compute "true" vertical displacements of air z_A using the observed balloon track z_B for two distinctly different flow patterns (figs. 4 and 5).

Flight number 3 of Jan. 19, 1967, shows a "hydraulic-jump-type" flow pattern with extended downslope winds and a subsequent sudden decrease in wind speed and lifting of air trajectories. The superimposed wave activity is only light. In this case, the restoring force brings the balloon back to the equilibrium level much too soon. The actual downdraft is strongly underestimated.

A little more than an hour later, a regular lee-wave pattern had established itself, and here the balloon followed the air motion quite well with only a slight phase lead. As these computations are only tentative and most balloon flights occurred in more-or-less regular wave flow, the rest of the balloon data is presented without corrections.

For the present investigation the procedure described by Booker and Cooper (1965) for ground launching via tow balloons was followed in essence. The launching station was in most cases upwind of Granby Lake, a large frozen reservoir west of the Front Range, while the tracking radar, an M–33, was located about 60 km almost due east, as shown in figure 1. In order to avoid ground clutter problems and allow for greater tracking range, instead of a radar reflector a transponder was hung from each balloon, returning a 300 mc signal which was then fed into the radar and tracked as if it were an ordinary reflected signal.

During the winter of 1966–67, 44 balloons were launched. Nineteen of these flights resulted in successful trajectory determinations (table 1). Some of the failures were due to human error or mechanical or electronic failures in the various components of the system. In the majority, however, ice and/or snow accumulation on the balloon in the cap cloud was the most probable cause. The small balloons used, although desirable for their large drag, could be flown with only about 50 gm of initial free lift. An accumulation of snow or ice greater than that would cause a balloon to sink, generally to the surface west of the mountain crest. Humidity conditions in westerly flow over Colorado vary over a substantial range, but in most cases the lifting over the Front Range is sufficient to produce a cap cloud extending a kilometer or more above the crest. This cap cloud usually produces light snow near the crest, but contains considerable amounts of supercooled water and has therefore often been considered a prime candidate for economically significant nucleation. Since a 15-micron coating of ice on the balloon skin (or 4 times that in water equivalent of dry snow on the top) is sufficient to bring down a balloon, it is not surprising in retrospect that few if any survived passage through the cap cloud. There were, in fact, no successful flights with floating altitudes less than 0.5 km above the crest (below 600 mb), while the highest possible floating altitude was limited to about 450 mb by balloon volume and weight. Because of this limitation and the generally lower reliability, lesser information content, and higher cost of the balloon flights as compared to aircraft, the use of balloons was discontinued after successful airplane flights had been demonstrated. Also, operational difficulties were greater than anticipated, that is, the time delay between decision and operations turned out to be at least 6–7 hr, marginally sufficient for lee wave patterns, but far too long for probing the sudden, unpredictable "chinooks" or downslope wind storms for which the Boulder area has acquired local notoriety.

Evaluation of the raw radar tracking data was straightforward. Range, azimuth, and elevation, as read off at 1-min intervals, were smoothed and converted into x, y, z positions and respective velocities. The graphical display unit of NCAR's CDC 6600 computer made it possible to plot all desired quantities (Z, U, W) against X, rather than elapsed time.

Figures 2 through 7 present balloon trajectories. Table 2 summarizes the characteristics of each flight.

Figure 8 contains a sample of profiles of the wind component u across the ridge. No special upstream soundings were taken in 1966–67. Denver soundings, released 70 km downwind from the Divide, are presented instead in figure 9. Distortion of these soundings by lee waves cannot be ruled out, but usually strong wave activity does not reach so far downstream. The nearest upstream soundings are available from Grand Junction, Colo., about 270 km west-southwest of the Divide.

3. AIRCRAFT

Radok (1954) was apparently the first to report on the use of powered aircraft to obtain quantitative data on mountain wave dynamics. The instrumentation available to him was fairly primitive by present standards. Our flights were made with instrumentation and methods still less than optimal, but generally similar to those of the ESSA Research Flight Facility aircraft in some reported flights through hurricane eye wall and spiral band clouds (Gray 1965). The airplane used here was a Beech Queen Air 80, provided by the NCAR Facilities Division and equipped with sensors and digital conversion equipment for measuring and recording temperature (reverse flow thermometer), static pressure, air speed (pitot tube), heading (magnetic compass), and ground speed and drift angle (Doppler navigation system). No humidity data were available. Most flights were made at constant engine power setting and constant attitude (pitch angle) so that to a first approximation, valid over wavelengths from a few to 20–40 km, the airplane's vertical velocity is the same as that of the air. The accuracy of this approximation is probably of the order of 1 m sec^{-1}. It varies somewhat with the pilot's skill and the severity of turbulence and other environmental factors, but can be largely determined by the constancy of the indicated air speed. In our flights the methods apparently improved with time as pilots became more experienced. At a later time it became evident that a more suitable flight mode for this type of aircraft was to maintain constant airspeed, and this mode was commonly used in the 1968 flights.

Measurement flights were made over the Front Range on 7 days, in February and March 1967. On most of these days one leg each eastward and westward was flown at altitudes of roughly 7.5, 6, and 5 km along the path indicated in figure 1. The direction of this path deviates slightly from the normal to the mountain range, but was determined so that the Doppler navigation data could be

TABLE 1.—*Operational data of all attempted balloon flights during the winter of 1966/67*

Date	Flight number	Local launch time	Launch site	Remarks	Success	Transponder returned from	Weather
Nov. 28, 1966	1	14:30	Marshall Field (Test)	No separation. Ruptured; falling down fast			
	2	16:23		Transponder bad?			
Dec. 2, 1966	1	15:33		Transponder not activated		Byers, Colo.	
	2	16:41		Good	X		
Dec. 8, 1966	1	9:33	Granby	Partially good	X	Foothills	Wet-looking cap cloud. Sfc. wind 5-10 kt at radar
	2	12:00		Good	X		Few lenticulars
	3	14:35		Not contacted			Light snow showers at launching site; heavy overcast
	4	16:14		Picked up; lost again			
Dec. 22, 1966	1	10:35		Picked up but crashed?			Mountains socked in, unstable, snow flurries E. of mountains.
	2	12:07		Picked up but crashed?			Launch site: overcast, light snow
Jan. 1/2, 1967	1	22:21		Not contacted			
	2	23:44		Not contacted			Light to moderate snow at launch site, NW. wind 15 kt
	3	1:07		Good	X		
Jan. 3/4, 1967	1	20:58		Not contacted			
	2	21:58		Not contacted			Intermittent light snow showers at launch site
	3	22:53		Picked up but went too high. Ruptured			SW. wind
	4	0:12		Picked up but went too high. Ruptured			
	5	1:12		Picked up but lost again		Rush, Colo.	
Jan. 4, 1967	1	11:04	Granby	Good long flight	X		Light snow at launching site, clearing soon
	2	12:23		Do.	X		Broken clouds
	3	14:33		Picked up initially but crashed (?)			
	4	15:28		Not contacted			
	5	16:28		Partially good. Came down 4 mi S. of Boulder	X		
	6	17:34		Lost on Divide			
Jan. 5, 1967	1	10:50		Picked up intially but lost on Divide			At launching site: overcast, no snow
	2	11:54		Ruptured because of too fast ascent?			Light snow, W. wind 15 kt
Jan. 19, 1967	1	10:39		Not picked up			
	2	11:33		Fast ascent (1,000 ft min^{-1}). Partially good	X	Denver	At launching site: sunny, clear, calm
	3	12:38		Good	X	Limon, Colo.	Ci, Ac, Ac lent., typical foehn sky; lots of waves all over the sky
	4	13:58		Good	X		
	5	15:17		Went up too high at first. Good	X		
	6	16:28		Separation observed, but not contacted			
Jan. 28, 1967	1	11:56	Granby	Fast ascent, ruptured, crashed			
	2	13:03		Good	X		
	3	14:27		Good	X	Lakin, Kans.	At launching site: sunny
	4	15:49		Good	X	Limon, Colo.	Ci, Cs, As, Ac lent. Peaks clearly visible. Wave clouds
	5	17:18		Good	X	Flagler, Colo.	
Jan. 30, 1967	1	17:35		Good	X	Brush, Colo.	At launching site: few scattered clouds
	2	19:06		Good	X		Cs, Ac lent., calm. Peaks visible
	3	20:25		Good	X		
	4	22:30		Not contacted			
Feb. 9, 1967	1	14:40		Partially successful	X	Fountain, Colo.	Broken clouds, Ac lent., Cu, Sc at Divide. Light SW. wind, peaks partly visible
	2	16:08		Not contacted			
Feb. 10, 1967	1	15:32		Not contacted		Denver	At launching site: broken clouds, cap cloud over mountain. NW. wind 15 kt, unstable and showery
Total: 44 launches, 19 successful					Eleven transponders returned.		

checked against the VORTAC navigation system located near the Denver commercial airport. The first legs were generally at the highest altitude, while the last could be as low as 1 km above the mountain crest depending on the pilot's judgment of the severity of turbulence, icing, visibility restrictions, etc.

The raw data output was processed through NCAR's CDC 6600. Assuming that the lateral displacement of the plane (at right angles to the heading) is due to a wind component only, the x- and y-components of the wind speed can be computed from geometric considerations, using the Doppler navigation information, indicated air speed, and heading. Instrumental errors in the Doppler system were partially removed by comparisons of eastward and westward flight-leg data.

The weakest link in the data, apart from the assumptions on vertical velocity, is the x,y-position of the airplane. Relative horizontal positions were obtained by

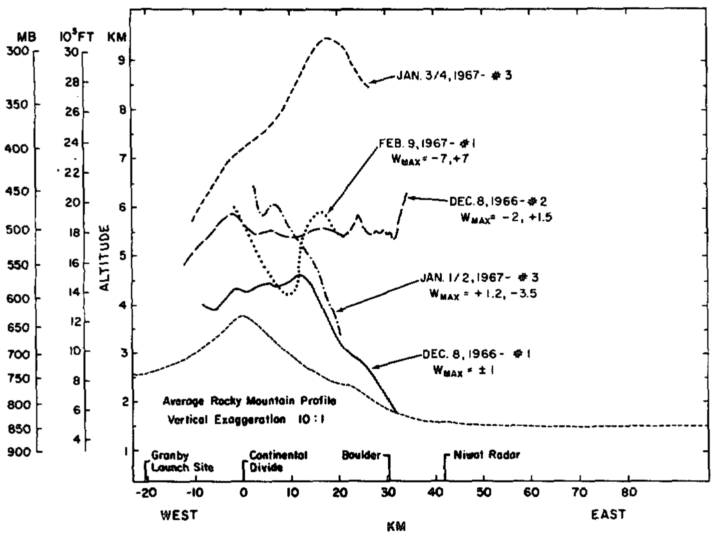

FIGURE 2.—Balloon trajectories in a west-east cross section for the indicated days and flights.

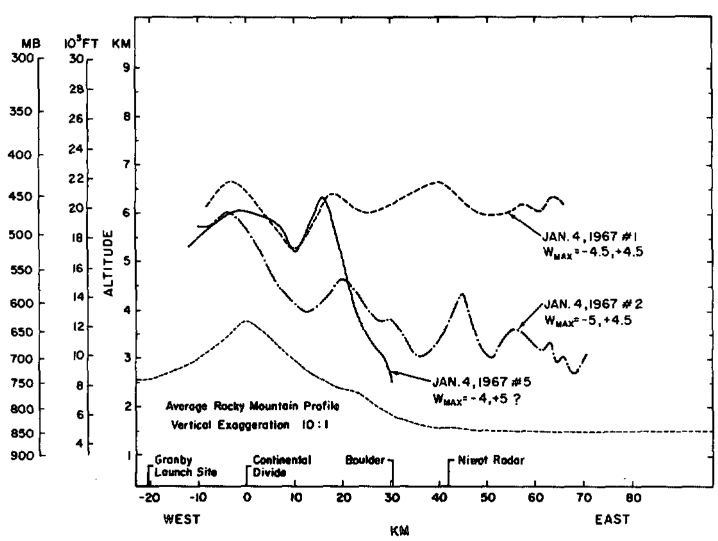

FIGURE 3.—Balloon trajectories in a west-east cross section for the indicated day and flights.

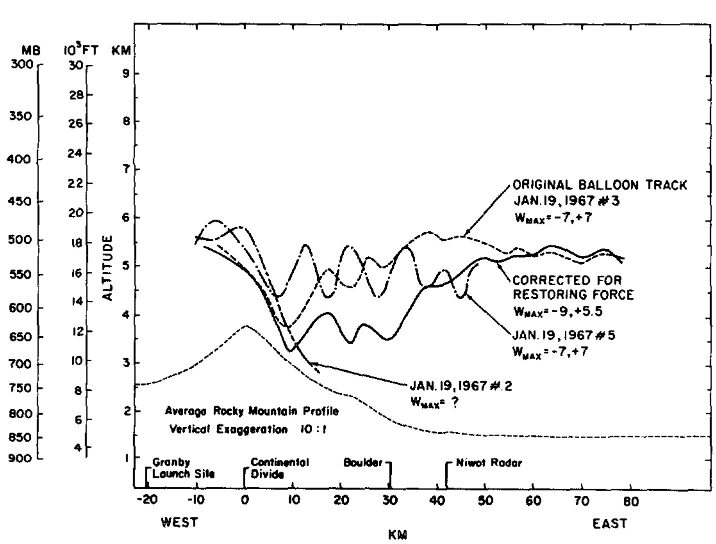

FIGURE 4.—Balloon trajectories in a west-east cross section for the indicated day and flights.

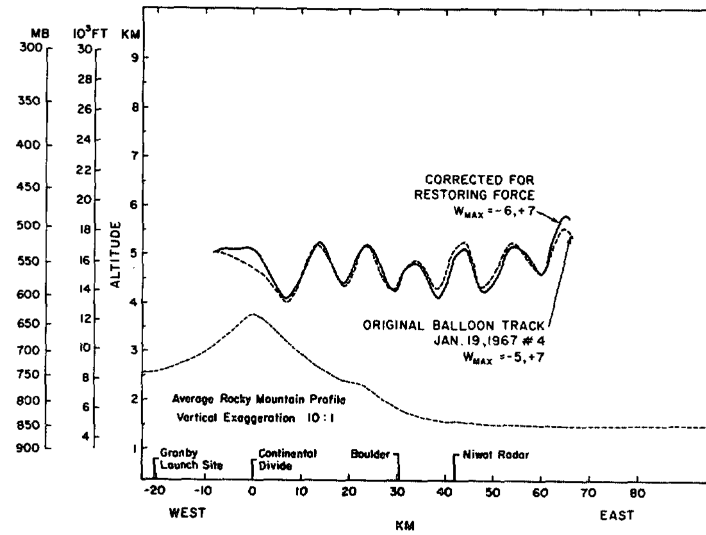

FIGURE 5.—Balloon trajectory in a west-east cross section for the indicated day and flight.

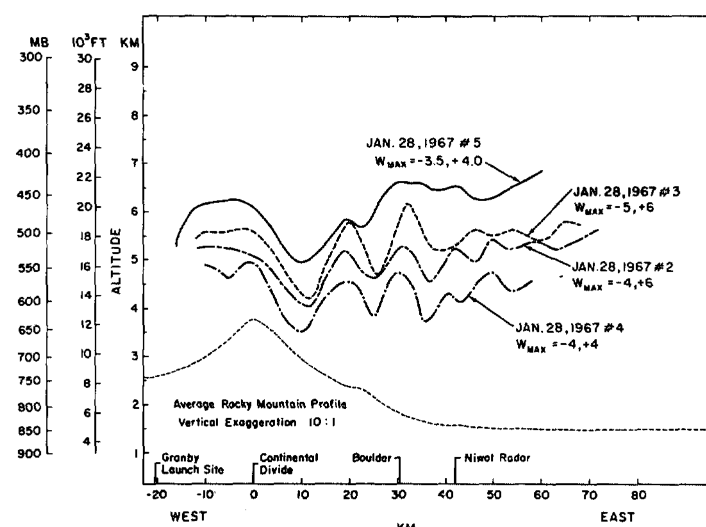

FIGURE 6.—Balloon trajectory in a west-east cross section for the indicated day and flights.

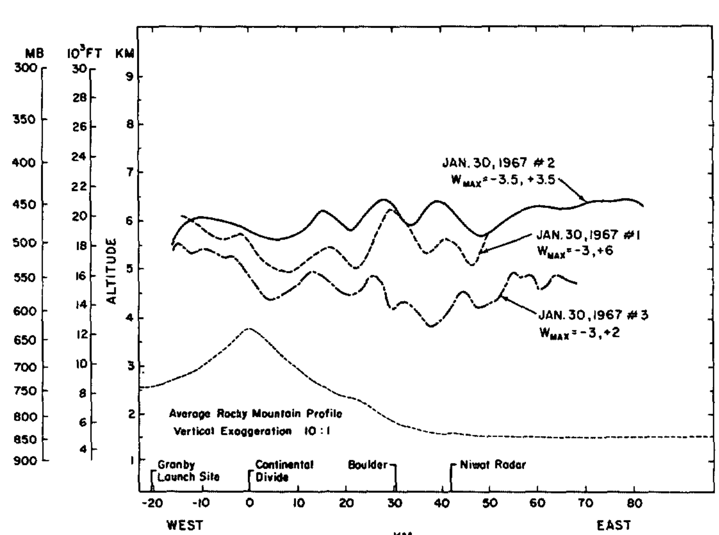

FIGURE 7.—Balloon trajectories in a west-east cross section for the indicated day and flights.

TABLE 2.—*Characteristics of the successful balloon flights in comparison with resonance modes computed from a lee wave model*

Date	Flight no.	Remarks	Mean wind along flight track: Direction 0(N.)-360°	Speed u (m sec^{-1}): Upstream	Speed u (m sec^{-1}): Downstream	Equilibrium flight level (km)	Wavelength (km)	Amplitude of streamline [2] (km)	w_0 [3] (m sec^{-1})	Resonance wavelengths computed from lee wave theory, amplitude of corresponding modes and their stability with respect to changes in upstream conditions
Dec.2, 1966	2	Test-balloon floats around inversion	145		−6	2.6				
Dec.8, 1966	1	Came down north of Boulder, rather regular wave, weak	280	20	13	4.5	6.8		1	
	2	Jump-type-2 sharp drops in u at $x=3$ km and at $x=24$ km, plus weak wave [1]	300	18	3	5.5	8.5	0.08	0.6	
Jan.1/2, 1967	3	Came down southwest of Boulder, mixed pattern	305	9	8	6.0	10	0.2?	1	
Jan.4, 1967	1	Jump-type and waves, interference	315	30	18	6.2	20	0.4	3	
	2	Jump-type and waves, interference	300	27	13	4.0	20 6?	0.55	3.5	
	5	Came down south of Boulder, rather regular wave	295	25	19	5.7	20	0.6	4	
Jan.19, 1967	2	Came down northwest of Boulder	280	28	?	5.7		$w_{max}=\pm 6$ m sec^{-1}		
	3	Jump-type with weak wave, sharp drop in u at $x=23$ km	290	30	17	5.3	11?	$w_{max}=\pm 7$ m sec^{-1}		
	4	Well-developed lee wave	305	26	15	4.7	10.4	0.40	5	10–11 km, strong, very stable, max around 500 mb
	5	Strong lee wave and sharp drop in u	305	27	15	4.9	10.4	0.55	7	
Jan.28, 1967	2	Damped lee wave and jump at $x=+23$ km	295	21	17	5.0	11.0	0.30	3	Around 20 km, medium, but sensitive. Max at tropopause
	3	Strong wave, damped downstream, interference	285	19	17	5.4	12.0	0.60	5.5	
	4	Lee wave damped downstream, interference, decrease of u downstream	290	19	12	4.2	12.0	0.50	4	Very strong, high modes, but extremely sensitive
	5	Lee wave, interference, no drop in u	290	21	18	6.0	11.5?	0.30	3.5	
Jan.30, 1967	1	Undamped wave, interference, no drop in u	265	21	20	5.5	14	0.35	3	
	2	Rather regular wave, no drop in u	265	23	23	6.1	13.5	0.30	3	15–25 km, strong, but sensitive. Max around 500 mb
	3	Weaker wave with much interference, strong decrease of u downstream	245	17	12	4.6	17? 6.5?	0.25?	2	
Feb.9, 1967	1	Strong jump at $x=11$ km, strong downdraft	325	33	11	5.6	$w_{max}=\pm 7$ m sec^{-1}			

[1] In this column, x is distance downstream from the Divide.
[2] Amplitude of streamline is half the total displacement.
[3] In this column, w_0 is the amplitude of the vertical motion (average), usually less than the primary downdraft.

time integration of the two components of the ground speed (Doppler radar), and they appear to be reliable. The only absolute position data, however, were four or five readings of the distance from the Denver VORTAC along each flight leg, and this dial could not be read accurately. The uncertainty in absolute x- and y-locations of all data points is estimated to be ± 2 km, decreasing for later flights, when the crew became more experienced. A lag of about 5 sec in the response of the temperature sensor was traced to an inadequate location of the sensor in a quasi-stationary eddy. The computer output, again facilitated by the graphical display unit DD80, includes static pressure, potential temperature, u and w (time derivative of static pressure, converted into m sec^{-1}) along each flight leg, plotted against x.

There are three possible ways of computing the two-dimensional flow field:

1) Potential temperature is plotted according to the current (x, z) position of the airplane, assuming that the flow is reasonably stationary and adiabatic, so that isentropes are streamlines. For most days this is the best method, particularly if the vertical profile of potential temperature is close to linear. If the basic flow has a sharp inversion, however, with a very small gradient outside of it, highly nonsinusoidal horizontal profiles of potential temperature may be measured on the airplane which cuts through the wavy inversion almost horizontally. The airplane will then experience a fixed temperature jump, and no more, every time it crosses the inversion, regardless of the wave amplitude. Thus the wave amplitude cannot be inferred from such traces. In addition, the problems of nonstationarity mentioned before render the use of potential temperature as a tracer difficult on some runs. Figure 19 gives an idea of the changes in basic stability that can occur within 6 hr.

2) Using $u(x)$ or $w(x)$ or both with surface wind data, one can compute the stream function. The horizontal wind speed $u(x)$ is more reliable than $w(x)$. The internal con-

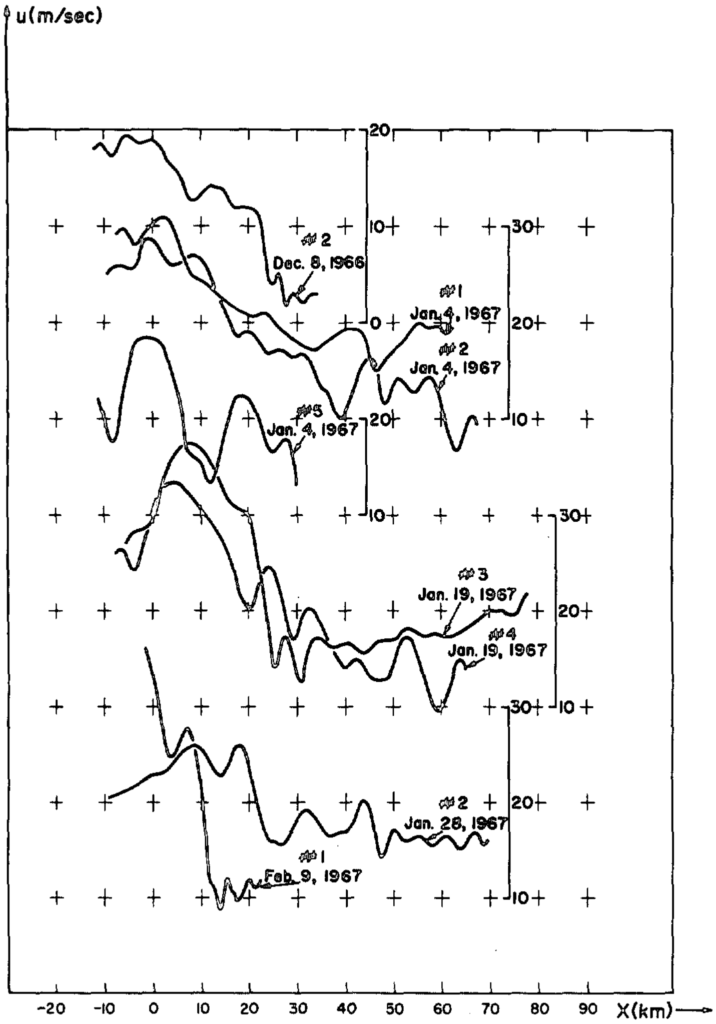

FIGURE 8.—West-east wind component u as a function of west-east distance x for the selected days and flights.

sistency of u and w was investigated by checking mass continuity. This, however, involves velocities at two flight levels, measured an hour or so apart, and introduces errors of uncertain magnitude due to nonstationarity. A program was written to modify the measured u- and w-velocities, assigning different "reliability" to u and w, in such a way that they would exactly satisfy mass continuity in each $(\Delta x, \Delta z)$ block, while deviating as little as possible from the original values. This procedure was adequate close to the lee slope where u and w had to be changed only slightly, but farther downstream the pattern was too incoherent between different runs, and more or less schematic streamlines had to be drawn. On 2 days the wave activity was too incoherent or three-dimensional for this method to be applied.

3) After assuming a stationary flow pattern with little vertical variation, in which the airplane moves with the vertical air motion, the height change of a true streamline Δz_A is related to the height change Δz_p of the airplane by $\Delta z_A = (u_p/u_A)\ \Delta z_p$ where u_A is the wind velocity and u_p is the ground speed of the airplane in a direction across the ridge. This relation was integrated for a few days, only one of which is presented here, and yields rather crude streamlines.

Table 3 gives a summary of all airplane flight data. Figures 10 through 17 show flow patterns derived by methods 1), 2), or 3), depending on the quality of data available. Selected horizontal profiles of wind speed u are presented in figure 18, and soundings in figure 19.

4. SURFACE ANEMOMETERS

In addition to the tropospheric data obtained by balloons and aircraft, about eight recording anemometers have been maintained in the Front Range-Boulder Valley area continuously during 1965–1969, located as shown in figure 1. The principal purpose of the anemometry was to record the chinook downslope wind storms, which are frequent and occasionally severe in this area, but whose mechanism is still poorly understood. In the analysis of flight data to be presented, the surface winds were used somewhat qualitatively to help establish the lower atmospheric wind structure. There is, however, an unfortunately large data gap of 2–3 km between the surface in the foothills and plains and the lowest flight level. Some possible means of filling this data void are discussed at the end of this paper.

5. MAIN FLOW FEATURES OBSERVED

Lee wave activity of some sort occurs in a variety of synoptic conditions in winter and spring, as long as the atmosphere is moderately stable and the wind speed has a sufficiently large component from the west, typically around 15–30 m sec^{-1} at 500 mb. Typical values are 7–20 km for wavelength (tables 2 and 3), 1 km for total streamline displacement, and 3–5 m sec^{-1} for vertical velocity. Considerably larger vertical velocities have been reported in the Sierra Wave Project (Holmboe and Klieforth 1957) presumably due to the resonance effect of the additional downwind mountain range. It appears, however, that as yet no instrumental data have been obtained on Front Range waves during days on which lee turbulence was considered severe by airways forecasters.

Two distinctly different flow patterns emerged from the flights: 1) regular *lee wave flow* with the mean wind u roughly constant downstream and waves extending at least 50–100 km downstream with little damping and 2) *hydraulic-jump-type flow* with a strong downdraft and high wind speed u immediately to the lee of the Divide, a sudden recovery of the streamlines with a sudden decrease in wind speed u (jump) and only weak wave activity thereafter.

Only rarely do those two types occur in pure form. A third type of flow could be added: irregular three-dimensional flow structure. It seems to be frequent with weak winds across the mountains or with the wind direction strongly from the north or south, but we had no crosswind resolution in our data to substantiate details.

Some patterns might be described in terms of either "long wave," roughly in phase with the mountain shape, or subcritical (Feb. 10, 1967, fig. 10) and supercritical flow (Feb. 14, 1967, fig. 15 and Mar. 2, 1967, fig. 14),

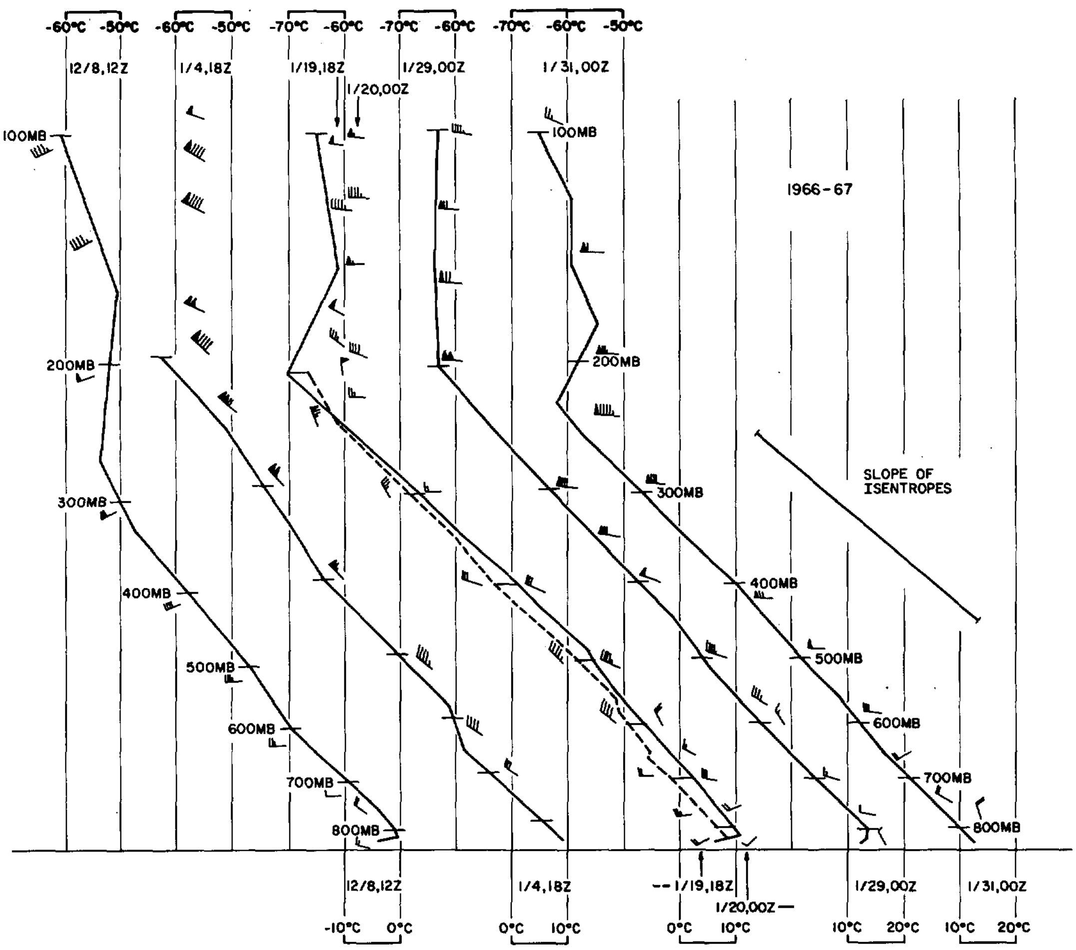

FIGURE 9.—Denver soundings for the indicated days and times. 00Z (GMT) corresponds to 17 hr MST (local time) of the previous day. 12Z corresponds to 05 hr MST of the same day.

respectively. Subcritical flow (low speed, streamlines dipping over the obstacle) and supercritical flow (high speed, streamlines bulging over the obstacle) are two possible regimes besides the hydraulic jump in shallow fluid theory. Strong surface winds accompanied by blowing sand and dust typically occur in the plains to the east of the foothills when the airflow resembles the supercritical pattern.

Nothing need be said at this point about the theoretical basis for lee wave flow. The concept of the "hydraulic jump," long familiar to engineers, was applied to the flow over obstacles by Long (1954) and Kuettner (1958) (see also Queney et al. 1960) for the first time. A complete theory based on the time dependent, nonlinear shallow water equations has recently been worked out by Houghton and Kasahara (1968) for a one-layer and by Houghton and Isaacson (1969) for a two-layer system. The qualitative agreement between some jump-type observations, best seen in the $u(x)$ profiles (figs. 8 and 18), and the theory is striking, although no attempt has been made to obtain quantitative agreement. The theory remains hard to apply to atmospheric data because it allows for a fluid system with one or two homogeneous layers only and because it is hydrostatic and therefore does not admit lee waves.

A strong motivation for Houghton and Kasahara's theoretical studies, as well as for this whole observational program, was the hope of explaining the violent chinook winds that strike the foothills of the Colorado Rockies in winter and spring. Nature was uncooperative, and no data were obtained in actual chinook conditions, either by balloon or by airplane.

The hydraulic-jump theory remains a strong candidate to explain, at least partly, the chinook in either of two ways: 1) as a "shooting stream," the strong downslope current associated with supercritical flow on the lee side of the obstacle, and 2) as a penetration of midtropospheric air to the surface on the lee side of the mountain, while the flow at lower levels upstream is blocked by the mountains. Blocking has long been recognized as a possible mechanism for the chinook (Scorer and Klieforth 1959, Beran 1966). It occurs in hydraulic-jump theory when the obstacle is high and the Froude number small.

6. THE STATIONARITY OF LEE WAVE FLOW

The assumption that the lee wave flow is stationary has proved exceedingly convenient both for theoretical studies and for data evaluation. Not surprisingly, our data show that considerable changes of the flow pattern occur even within 30 min or 1 hr, although a resonance

TABLE 3.—*Characteristics of the airplane flights in comparison with computed resonance modes*

Date, time	Run no., heading	Remarks	Mean wind along flight track: Direction	Speed u (m sec^{-1}): Upstream	Speed u (m sec^{-1}): Downstream	Mean flight level (km)	Wavelength (km)	Computed amplitude of streamline (km)	w_0 (m sec^{-1})	Max potential temp. variation	Resonance wavelengths, computed from lee wave theory, amplitude of corresponding modes, and their stability with respect to changes in upstream conditions
Feb. 10, 1967 14:30–18:15	1 W.	Long, regular wave	300	26	24	6.3	45	0.56	1.8	3.5	
	1 E.	Long wave	300	25	25	6.2	60?			1.5	
	2 W.	Incoherent, weak waves	310	21	25	5.6	40?	0.22	0.7	1.5	
	2 E.	Incoherent, weak waves	310	22	22	5.6				0.6?	Weak, unstable modes
	3 W.	Long wave?	315	13	13	4.9	45?			1.2	
	3 E.	Long wave?	315	8	8	4.6	30?			2.0	
	4 W.	Small jump and weak waves	325	12	8	4.5				3.0	
	4 E.	Long wave	325	6	6	4.2	60			1.5	
Feb. 12, 1967 11:25–14:15	1 W.	Waves with interference	310	27	25	7.8	40? 12?	0.85	2.5?	3.5	Approx. 12 km, strong, stable, max in lower troposphere
	1 E.	Waves with interference	310	24	23	7.8	40? 12?	0.85	2.5?	3.0	
	2 W.	Jump at Divide + long wave + shorter waves	325	18	13	6.2	40	0.30	1.2?	1.5	
	2 E.	Jump at Divide + interfering waves	325	17	11	6.2	25?	0.30	1.2?	1.0	15 km, medium, very stable, max at tropopause
	3 W.	Jump behind Divide + waves	325	21	14	4.8	30?	0.65	1.5?	4.0?	
	3 E.	Jump behind Divide + waves	325	16	11	5.1	30?	0.65	1.6?	3.5?	
Feb. 13, 1967 10:40–13:45	1 W.	Long + short waves	270	20	18	7.1	35 13?	0.55	2.5?	3.0	Approx. 10 km, strong, stable, max in lower troposphere
	1 E.	Interfering waves	270	20	17	7.1	25?	0.35	2.5	3.0	
	2 W.	Jump behind Divide, downdraft to 13 km E. of Divide	270	19	12	5.8	w(max) = −3, +4 m sec^{-1} Max displacement = 1.8 km			5.5	
	2 E.	Jump at Divide + waves	270	19	12	5.6	22	0.70	3.0	3.0	
	3 W.	Jump behind Divide + damped waves	270	18	18	5.0	w(max) = −4, +5 m sec^{-1} 15? Max displacement = 2 km			6.5	18 km, weak, unstable, max at tropopause
	3 E.	Jump behind Divide + damped waves	270	17	19	4.7	15? w(max) = −4, +6 m sec^{-1} Max displacement = 1.5 km			5.0	
Feb. 14, 1967 13:45–17:15	1 W.	Long wave	255	40	33	6.9	70	0.33?	1?	3.0?	
	1 E.	Long wave + drop of u downstream	255	38	32	7.0	75?	0.22?	1?	3.0?	
	2 W.	Long wave + short waviness decrease of u downstream	250	39	26	6.0	70 22?	0.44?	1.5?		50–70 km, strong, rather stable, max around 500 mb
	2 E.	u decreases downwind.	250	37	26	6.3	23?			3.0?	
	3 W.	Irregular wave	250	35	33	5.2	60? 20?	0.32?	2?	2.0?	
	3 E.	Ill-defined waviness	250	27	?	4.7	35?			2.0?	
Mar. 1, 1967 12:45–16:00	1 W.	Jump + large wave, damped	295	28	24	7.5	14.5	0.39	4.0	4.0	
	1 E.	Jump + large wave	295	31	26	7.5	16.5 w(max) = 6 m sec^{-1}	0.28	3.0	2.5	
	2 W.	Jump + large wave	295	28	24	6.4	16.5 w(max) = −6, + 9 m sec^{-1}	0.53	5.0	9.0	13 km, rather strong, very stable, max in troposphere.
	2 E.	Jump + large wave	295	28	24	6.6	17.5	0.48	4.5	2.5	
	3 W.	Jump + large wave	280	30	22	5.7	17.0	0.85	7.0	9.0	
	3 E.	Wave damped?	280	26	?	4.8	17?	0.85?	8.0	7.0	
Mar. 2, 1967 10:35–13:05	1 W.	Long + short, irregular waves	270	38	38	6.3	75?	0.70?	2?	6.0	
	1 E.	Long wave, ill-defined	270	40	42	6.4	60?	0.70?	1.5?		
	2 W.	Long wave, short waviness	265	26	26	4.6	80?	0.70?	2.0?	7.0	Weak, sensitive modes
	2 E.	Mixed waviness	265	29	30	5.0			3.0?		
Mar. 28, 1967 13:05–16:35	1 W.	Jump behind the Divide, downdraft extends 20 km E. of Divide.	260	38	29	7.8	w(max) = −1, + 2 m sec^{-1} Max displacement = 1 km			3.5	
	1 E.	Jump behind the Divide + waviness	260	41	31	7.9	23?	0.20?	1.5	2.5	
	2 W.	Mixed waviness, downdraft extends 13 km E. of Divide.	265	27	30	6.2	45? w(max) = −3, + 4 m sec^{-1} 13? max displacement = 1 km			5.0	20–25 km, strong, but very sensitive, max around 500 mb
	2 E.	Mixed waviness	265	29	31	6.2	25	0.30	2.5	2.0	
	3 W.	Long wave	250	22	19	4.7	30? w(max) = ±4 m sec^{-1} Max displacement = 1 km			4.5	
	3 E.	Well-defined, turbulent jump behind the Divide	255	28	14	5.0	Max displacement = 0.5 km?			1.5	

wave, once established, tends to approximately maintain its wavelength for a few hours. Tables 2 and 3 give an impression of the time variability of the flow. Figures 4 and 5 show two balloon trajectories (numbers 3 and 4) for Jan. 19, 1967, mentioned before. The flow changed from jump-type to wave-type within 1 hr. Anemometer observations of strong downslope winds commonly show rapid fluctuations of the location and intensity of the maximum surface flow.

On the highest airplane traverses, around 380 mb, patterns were never stationary between the westward and the eastward heading flight legs. Even well-developed

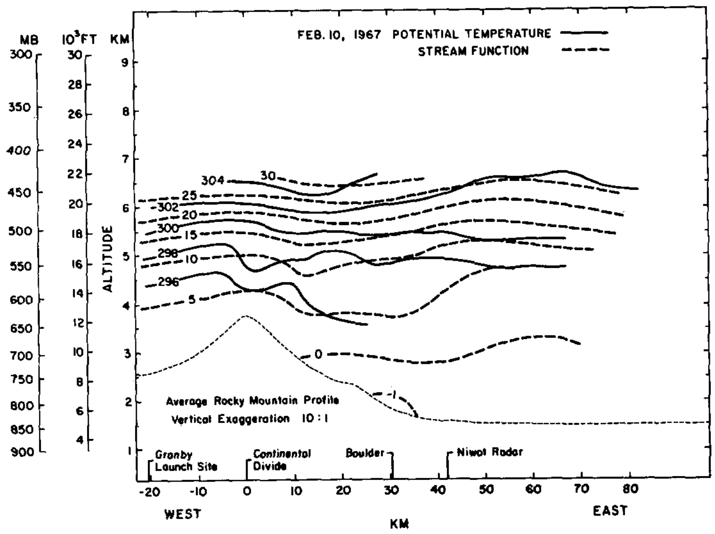

FIGURE 10.—Isolines of stream function ψ defined by $\rho u/\rho_0=\partial\psi/\partial z$, $\psi=0$ at lower boundary, and $\rho w/\rho_0=-\partial\psi/\partial x$ in units (m sec^{-1}) · km and isentropes (°K) for Feb. 10, 1967.

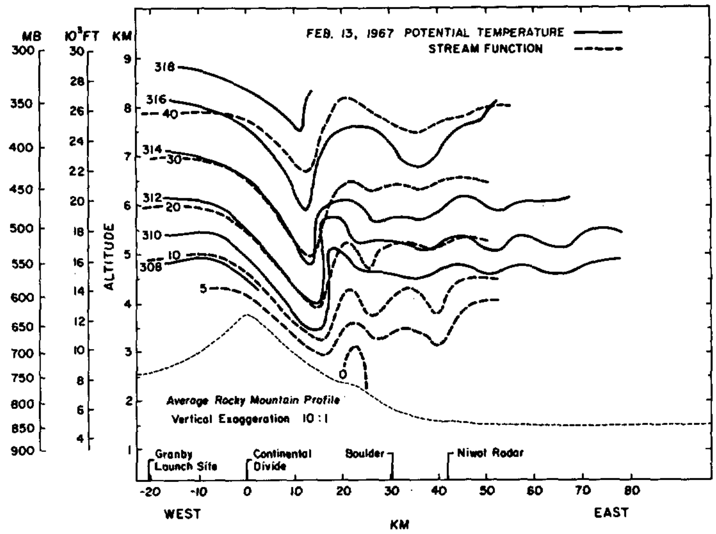

FIGURE 12.—Same as figure 10, except for Feb. 13, 1967.

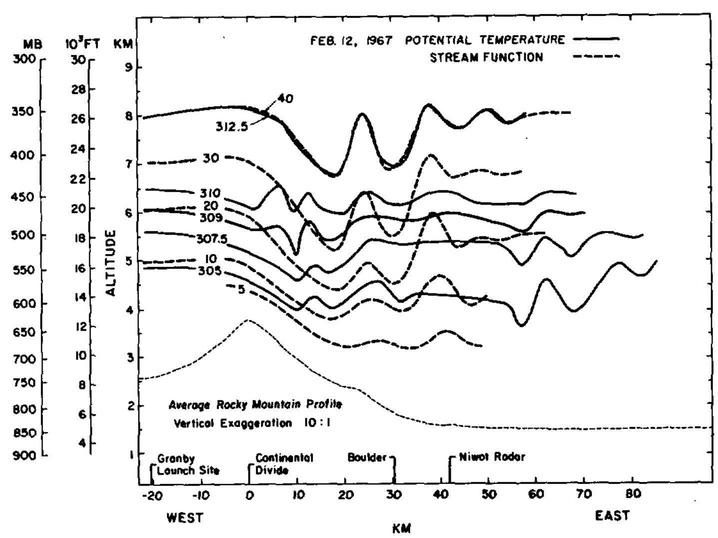

FIGURE 11.—Same as figure 10, except for Feb. 12, 1967.

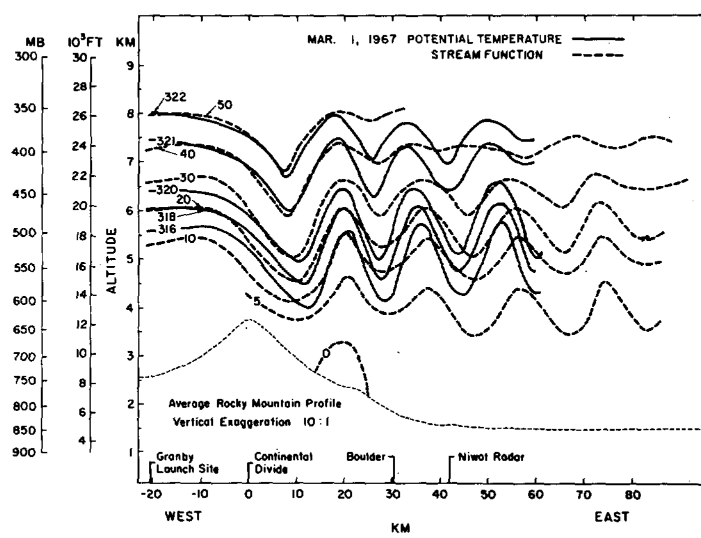

FIGURE 13.—Same as figure 10, except for Mar. 1, 1967.

regular lee waves as on Jan. 19, 1967 (figs. 4 and 5), or Mar. 1, 1967 (fig. 13), would tend to change phase and amplitude at a given altitude after two or three wavelengths.

A glimpse at the soundings, figure 9 and particularly figure 19, makes it obvious that rather substantial changes in the upwind flow structure within 6 hr are possible. Location, extent, and detailed structure of inversions in the sounding are changing most strongly. From classical lee wave theory there is ample reason to expect that the resulting flow pattern can be very *sensitive* to small changes in the upstream conditions (Corby and Wallington 1956), although this point of view has not been fully expounded. A great number of computations were performed by one of the authors with an operational linear model, using real input data. The model is essentially similar to Wallington's (1970). Special emphasis was placed, however, on determining the sensitivity of the resonance modes to small changes in the upstream conditions. It was found that the dominant resonance modes were very stable and corresponded well with observations (tables 2 and 3); but almost always, highly sensitive modes were present, too, sometimes with large nominal amplitudes, for which the assumption of stationarity is apparently not meaningful. Similar results were obtained by Danielsen and Bleck (1970). Wave computations using a single upstream profile can thus be misleading.

Regarding our data, it is impossible to say just how much time variation should be attributed to the changing upstream flow. However, many observed changes in the flow patterns appear to be too fast and too large to be caused in an accountable way by variations of the basic flow. These data, together with the computational experiments on sensitivity mentioned above, support previous notions on the possible nonstationarity (nonuniqueness) of the flow over obstacles (Long 1953). The bore propagating upstream and the jump propagating down-

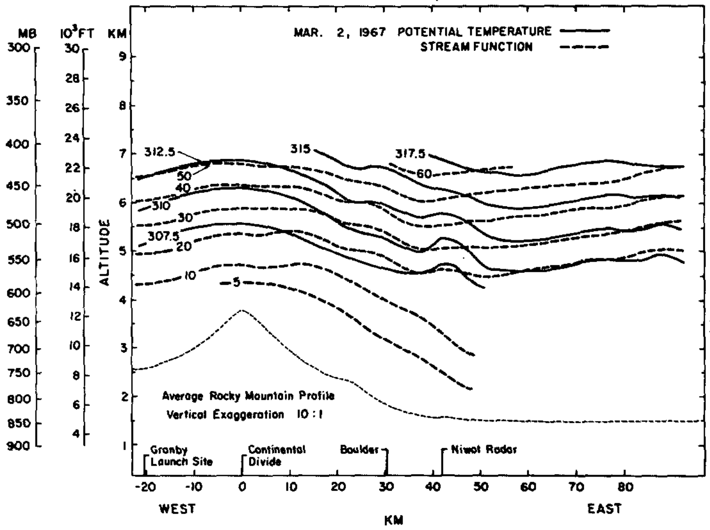

FIGURE 14.—Same as figure 10, except for Mar. 2, 1967.

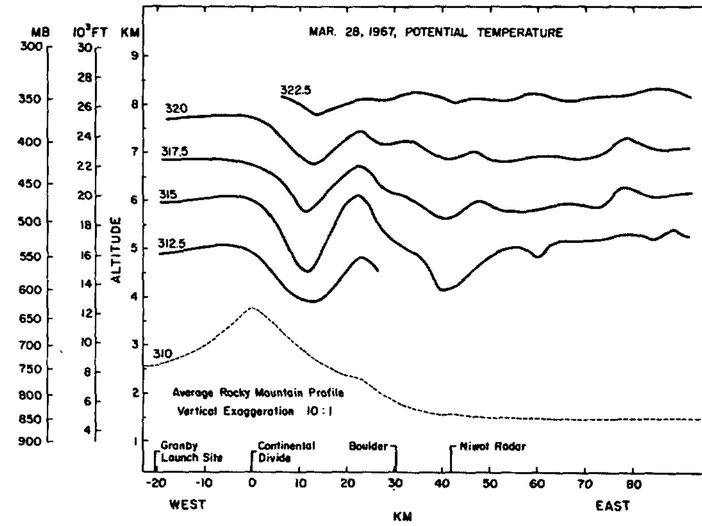

FIGURE 16.—Same as figure 15, except for Mar. 28, 1967.

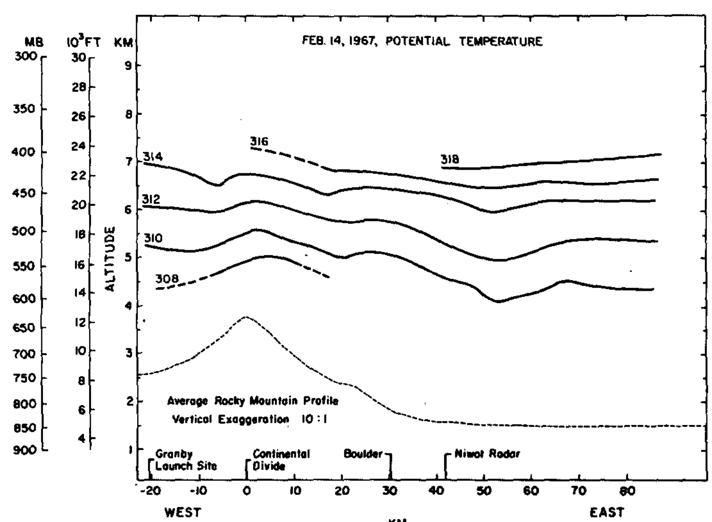

FIGURE 15.—Isentropes (°K) for Feb. 14, 1967.

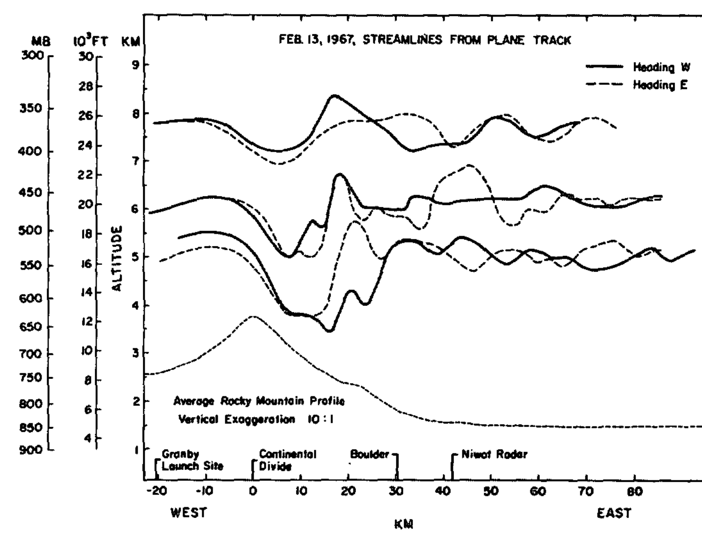

FIGURE 17.—Crude streamlines computed by method 3), see section 3.

stream in hydraulic jump theory are examples of such nonstationarity. Theoretical work needs to be done on how the sensitivity of computed resonance modes using a stationary model relates to the actual time-dependent flow.

7. CORRELATIONS AND FLUXES

Correlations $\overline{u'w'}$, $\overline{u'\Theta'}$, and $\overline{w'\Theta'}$ as well as others were computed for individual aircraft flight runs. They fluctuated rather wildly, but on the average $\overline{u'w'} \approx -1$ or -2 m^2 sec^{-2} (always negative) corresponding to a correlation between u' and w' of about 0.1 or 0.2 and $\overline{w'\Theta'} \approx 0$. Here, u', w', and Θ' are perturbation horizontal velocity, vertical velocity, and potential temperature. From the present data it does not seem feasible to verify the classical result of the constancy of momentum flux with height. This is not too surprising, as the main contribution to the momentum flux $\overline{\rho u w}$ comes from the primary downdraft (in other words, the continuous spectrum), while the lee waves (the line spectrum) whose energy is almost perfectly trapped do not effectively transport momentum or energy. The waves consequently act as noise for the purpose of flux calculations. Bretherton (1969) has recently pointed out the difficulties and ambiguities associated with flux computations.

For a single resonance wave it can be shown that $\overline{u'\Theta'}$ is proportional to $(d/dz)(\overline{\rho w_0})^2$ where w_0 is the amplitude of vertical velocity. This correlation should therefore be a help for finding the level of maximum wave activity. Sometimes it works well; at other times the interpretation of $\overline{u'\Theta'}$ is not clear, presumably due to multiple waves and/or nonstationarity. More sophisticated measurements of vertical velocity should improve the accuracy of the fluxes, although nonstationarity remains a problem.

8. OUTLOOK FOR FUTURE OBSERVATIONAL PROGRAMS

Airplane traverses proved to yield more and better information on lee wave flow than balloons. For future

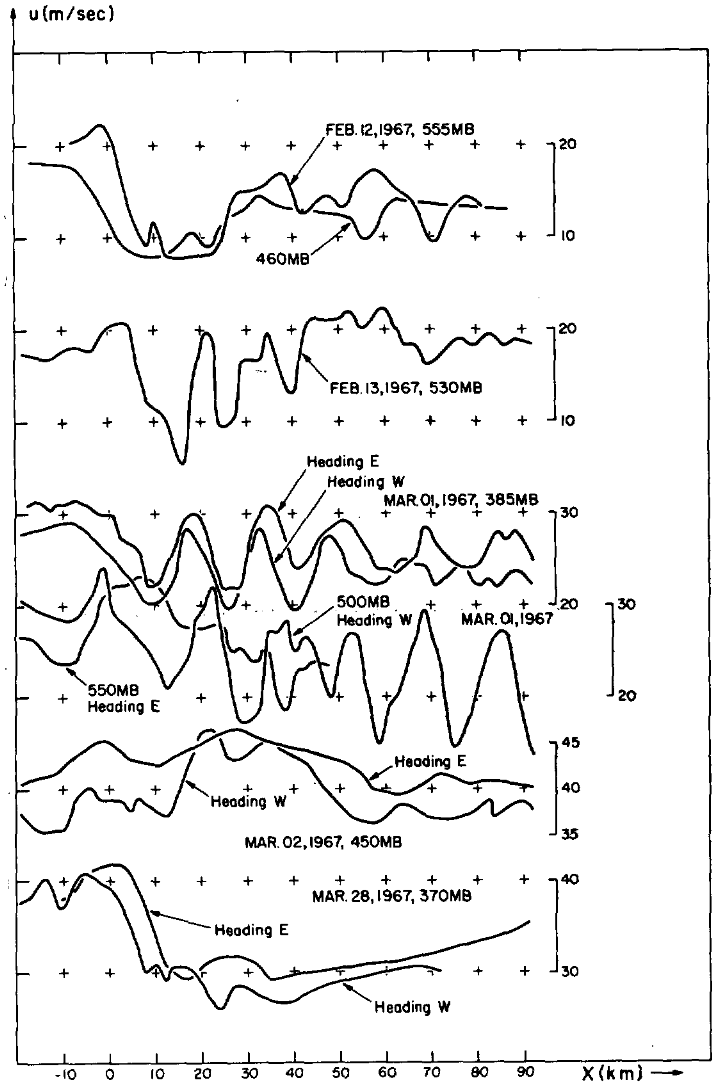

FIGURE 18.—West-east wind component u as a function of west-east distance x for the selected days and flights.

programs, a variety of other possibilities are considered which would fill the data gap in the lowest 3 km and, hopefully, be more flexible operationally:

1) surface temperature, pressure, and wind records on the upwind slope, particularly valuable for detecting presumed blocking effects in chinooks;

2) radar chaff released from a plane or shot up from the ground, serving as a target for Doppler radar;

3) other tracers, natural (ozone, aerosols) or artificial, released upstream and measured either by airplane or sailplane or by remote sounding techniques to be developed for detecting tracers and turbulence;

4) transponders or dropsondes carried by balloons and/or dropped on parachutes on the lee side, tracked by conventional radar;

5) Jimspheres tracked by FPS–16 radar for probing the detailed wind structure, including turbulence; and

6) parafoils attached to the ground by a cable and kept floating by the wind. This quasi-stationary probe, if practicable, would be attractive for investigating the sudden outbursts of chinook.

9. CONCLUSIONS

Quantitative observations of lee flow over the Colorado Rockies were conducted in 1966/67 using both constant-volume balloons and powered aircraft. Because of severe icing conditions and other operational difficulties, the balloon method was found to be significantly inferior to the use of instrumented aircraft.

From the usable data, examples of various kinds of lee flow regimes were observed, including smooth (subcritical or supercritical), wavy, jumplike, and irregular or unsteady combinations. Comparisons of some cases with linear numerical solutions suggest that when waves are predicted to be sensitive to small changes in the upstream environment, the observed patterns tend to be erratic and nonstationary.

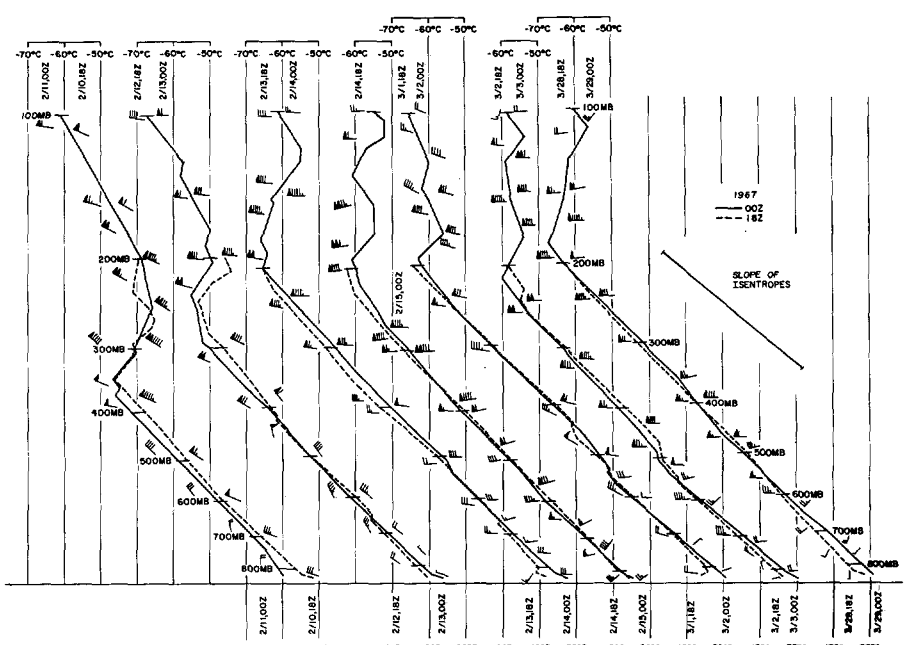

FIGURE 19.—Denver soundings for the indicated days and times. See figure 9 for further details.

ACKNOWLEDGMENTS

Many people were involved in various steps of the program. The NCAR Facilities Division supplied the M-33 radar with operators, the balloon launching van, and the Queen Air plane with instrumentation and pilots. Donald H. Lenschow was responsible for part of the instrumentation. Western Scientific Services, Fort Collins, Colo., participated in balloon launchings and radar operation. Karl Danninger served as observer on the airplane. Nancy Werner and other NCAR programmers helped to process the data through the NCAR computer. Graphics and printing were done by the respective NCAR facilities. Laura Potter typed most of the manuscript. One of the authors (I. Vergeiner) expresses his appreciation to all concerned for being able to participate in the program, primarily to Douglas K. Lilly of NCAR, Elmar Reiter, Colorado State University, Fort Collins, and to H. Hoinkes, University of Innsbruck, Austria, for granting the leave of absence.

REFERENCES

Beran, Donald W., "Large Amplitude Lee Waves and Chinook Winds," *Atmospheric Science Technical Paper* No. 75, Department of Atmospheric Science, Colorado State University, Fort Collins, Mar. **1966**, 92 pp.

Booker, D. Ray, and Cooper, Lynn W., "Superpressure Balloons for Weather Research," *Journal of Applied Meteorology*, Vol. 4, No. 1, Feb. **1965**, pp. 122–129.

Bretherton, F. P., "Momentum Transport by Gravity Waves," *Quarterly Journal of the Royal Meteorological Society*, Vol. 95, No. 404, Apr. **1969**, pp. 213–243.

Corby, G. A., and Wallington, C. E., "Air Flow Over Mountains: The Lee Wave Amplitude," *Quarterly Journal of the Royal Meteorological Society*, Vol. 82, No. 353, July **1956**, pp. 266–274.

Danielsen, E. F., and Bleck, R., "Tropospheric and Stratospheric Ducting of Stationary Mountain Lee Waves," National Center for Atmospheric Research, Boulder, Colo., **1970**, 28, pp., (unpublished manuscript).

Gray, William M., "Calculations of Cumulus Vertical Draft Velocities in Hurricanes From Aircraft Observations," *Journal of Applied Meteorology*, Vol. 4, No. 4, Aug. **1965**, pp. 463–474.

Harrison, Henry T., and Sowa, Dan F., "Mountain Wave Exposure on Jet Routes of Northwest Airlines and United Air Lines," *UAL Meteorology Circular* No. 60, United Air Lines, Inc., Feb. 1, **1966**, 66 pp. and appendices.

Holmboe, Jörgen, and Klieforth, Harold, "Investigations of Mountain Lee Waves and the Air Flow Over the Sierra Nevada," *Final Report*, Contract No. AF19(604)-728, Department of Meteorology, University of California, Los Angeles, Mar. **1957**, 290 pp.

Houghton, David D., and Isaacson, E., "Mountain Winds," *Studies in Numerical Analysis*, Siam Publications, Philadelphia, Pa., **1969**, pp. 21–52.

Houghton, David D., and Kasahara, Akira, "Nonlinear Shallow Fluid Flow Over an Isolated Ridge," *Communications on Pure and Applied Mathematics*, Vol. 21, No. 1, Interscience Publishers, New York, Jan. **1968**, pp. 1–23.

Kuettner, Joachim P., "The Rotor Flow in the Lee of Mountains," *Schweizer Aero-Revue*, Vol. 33, No. 4, Berne, Apr. **1958**, pp. 208–215.

Kuettner, Joachim P., and Lilly, D. K., "Lee Waves in the Colorado Rockies," *Weatherwise*, Vol. 21, No. 5, Oct. **1968**, pp. 180–197.

Lilly, Douglas K., and Toutenhoofd, Willem, "The Colorado Lee Wave Program," *Proceedings of the Symposium on Clear Air Turbulence and Its Detection, Seattle, Washington, August 14–16, 1968*, Plenum Press, New York, **1969**, pp. 232–245.

Long, Robert R., "Some Aspects of the Flow of Stratified Fluids: I. A Theoretical Investigation," *Tellus*, Vol. 5, No. 1, Feb. **1953**, pp. 42–58.

Long, Robert R., "Some Aspects of the Flow of Stratified Fluids: II. Experiments With a Two-Fluid System," *Tellus*, Vol. 6, No. 2, May **1954**, pp. 97–115.

Queney, Paul, Corby, G. A., Gerbier, N., Koschmieder, H., and Zierep, J., "The Air Flow Over Mountains: Report of a Working Group of the Commission for Aerology," *WMO Technical Note* No. 34, World Meteorological Organization, **1960**, 135 pp.

Radok, Uwe, "A Procedure for Studying Mountain Effects at Low Levels," *Bulletin of the American Meteorological Society*, Vol. 35, No. 9, Nov. **1954**, pp. 412–416.

Reiter, Elmar R., and Foltz, Harry P., "The Prediction of Clear Air Turbulence Over Mountainous Terrain," *Journal of Applied Meteorology*, Vol. 6, No. 3, June **1967**, pp. 549–556.

Scorer, Robert S., and Klieforth, Harold, "Theory of Mountain Waves of Large Amplitude," *Quarterly Journal of the Royal Meteorological Society*, Vol. 85, No. 364, Apr. **1959**, pp. 131–143.

Wallington, C. E., "Numerical Computation of Lee Wave Flow Including Rotors," *Aero Revue*, Basel-Zuerich, **1970**, (to be published).

[*Received September 5, 1969*]